Calculus Honors Summer Work

I. Rules for Exponents

	Rule	Example
1	$x^1 = x$	$5^1 = 5$
2	$x^0 = 1$	$5^0 = 1$
3	$x^{-1} = \frac{1}{x^1}$	$5^{-1} = \frac{1}{5}$
4	$(x^m)(x^n) = x^{m+n}$	$(x^2)(x^3) = x^{2+3} = x^5$
5	$\frac{x^m}{x^n} = x^{m-n}$	$\frac{x^3}{x^2} = x^{3-2} = x^1$
6	$(x^m)^n = x^{(m)(n)}$	$(x^3)^2 = x^{(3)(2)} = x^6$
7	$(xy)^n = x^n y^n$	$(xy)^3 = x^3 y^3$
8	$(\frac{x}{y})^n = \frac{x^n}{y^n}$	$(\frac{x}{y})^3 = \frac{x^3}{y^3}$
9	$x^{-n} = \frac{1}{x^n}$	$x^{-2} = \frac{1}{x^2}$

Simplify each expression below using exponent rules. Your final answer should not include any negative exponents. You MUST show work in order to receive credit.

1. $x^5 \bullet x^2$	2. $y^3 \bullet y \bullet y^4$	3. $b^4 \bullet b^{-4}$
4. $7x^3y^2 \bullet 5xy^9$	5. $a^{10} \bullet a^2 \bullet a^{-6}$	6. $(z^5)^5$

7. $\left(b^{7}\right)^{2}$	8. $\left(m^{-8}\right)^{-3}$	9. $\left(x^{2}y^{4}m^{3}\right)^{8}$
10. $\left(3x^{2}\right)^{4}$	11. $(2ab)^{5}$	12. $\left(2x^{3}y\right)^{6}$
13. $\left(m^{7}\right)^{4} \bullet m^{3}$	14. $p^{2} \bullet \left(p^{5}\right)^{2}$	15. $\frac{x^{5}}{x^{2}}$

II. Radicals and Fractional Exponents

Video to help: https://www.youtube.com/watch?v=8HfWGjQNlhk

When using fractional exponents, remember that the numerator is the power and the denominator is the root.

$$\sqrt[n]{x^m} = \sqrt[n]{(x)^m} = \left(\sqrt[n]{x}\right)^m$$

Example $27^{\frac{2}{3}}$

Numerator first $= \sqrt[3]{(27)^2} = \sqrt[3]{729} = 9$

Denominator first $= \left(\sqrt[3]{27}\right)^2 = (3)^2 = 9$

Write in simplest radical form. (Using only one radical sign in each problem

1. $64^{1/2} =$

2. $8^{-1/2} =$

3. $25^{3/2} =$

4. $x^{1/2} \bullet x^{1/2} =$

5. $a^{3/5} \bullet a^{1/5} =$

6. $k^{2/3} \bullet k^{-1/3} =$

III Dividing Fractions with a Monomial Denominator

Example: Divide: $\frac{x^3+3x^2-2x+5}{x}$

$$\frac{x^3}{x} + \frac{3x^2}{x} - \frac{2x}{x} + \frac{5}{x}$$

$$x^2 + 3x - 2 + \frac{5}{x}$$

Divide:

1. $\frac{5x^6-3x^4-4x^2+3}{x^2}$

2. $\frac{x^3-2x^2+4x}{\sqrt{x}}$ (hint: $\sqrt{x} = x^{\frac{1}{2}}$)

IV. Factoring

Factor:

1. $x^2 - 4$

2. $x^2 + 7x + 12$

3. $3x^2 + 10x + 3$

Factoring the GCF with Negative and Fractional Exponents

To factor an expression with the same variable to different powers:

1. Factor out the term with the lowest power;

2. Divide all terms by this factor to obtain the other factor;

3. Multiply the two factors together.

Negative exponents example

$$x^{-5} + x^{-6} = x^{-6}\left[\frac{x^{-5}}{x^{-6}} + \frac{x^{-6}}{x^{-6}}\right] = x^{-6}(x+1) = \frac{1}{x^{6}}(x+1)$$

Fractional Exponents Example:

$$x^{2/3} + x = x^{2/3} + x^{3/3} = x^{2/3}\left[\frac{x^{2/3}}{x^{2/3}} + \frac{x^{3/3}}{x^{2/3}}\right] = x^{2/3}(1 + x^{1/3})$$

For each of the following expressions: Factor out the common term with the lesser power and write in factored form. Write your answers with positive exponents only.

1. $x^{-5} + x^{-6}$

2. $x^{-7} + x^{-8}$

3. $x^{-8} + x^{-11}$

4. $x^{-7} + x^{-15}$

5. $\frac{1}{x^5} + \frac{1}{x^6}$

6. $\frac{1}{x^7} + \frac{1}{x^8}$

7. $\frac{1}{x^4} + \frac{1}{x^3}$

8. $\frac{1}{x^5} + \frac{1}{x^7}$

9. $x^{-2} + x^{-5} - x^2$

10. $x^{-4} + x^{-7} - x^2$

11. $x^{-3} + x^{-7} - x^3$

12. $x^{-5} + x^{-7} - x^4$

13. $x^{2/3} + x$

14. $x^{4/5} + x^2$

15. $x^{2/3} - x^{3/4}$

16. $x^{4/5} - x^{5/6}$

ANSWERS TO ODD QUESTIONS:

1. $\frac{1}{x^6}(x+1)$

3. $\frac{1}{x^{11}}(x+1)(x^2-x+1)$

5. $\frac{1}{x^6}(x+1)$

7. $\frac{1}{x^4}(1+x)$

9. $\frac{1}{x^5}(x^3+1-x^7)$

11. $\frac{1}{x^7}(x^4+1-x^{10})$

13. $x^{2/3}(1+x^{1/3})$

15. $x^{2/3}(1-x^{1/12})$

V. Lines

Slope Formula: $m = \frac{y_2 - y_1}{x_2 - x_1}$

Point-Slope Form of the Equation of a line: $y = m(x - x_0) + y_0$

Example 1. Find the equation of the line in point-slope form with the given conditions:

a) A slope of 5; through the point (-1, 2)

b) Through the points (-2, 3) and (-1, 7)

c) Through the point (4, 8) with an undefined slope

d) Through the point (4, 8) with a slope of 0

e) Parallel to the line y= 3x-2 through the point (1. 9)

f) Perpendicular to the line y=3x-2 and through the point (1, 9)

VI. Trigonometry

Video to help:

https://www.youtube.com/watch?v=1I7Jp62sGXM&nohtml5=False

Fill out the table below. Use it and "All Students Take Calculus" to evaluate all six trigonometric functions at the given angle. Be sure to sketch the angle in the correct quadrant in order to designate the appropriate sign. **THESE MUST BE MEMORIZED!**

ϑ (degrees)	0°	30°	45°	60°	90°	180°	270°
ϑ (radians)	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$
$\sin\vartheta$							-
$\cos\vartheta$							
$\tan\vartheta$							

Find the exact value without the use of a calculator:

1. $\cos\frac{\pi}{6}$
2. $\sin\frac{\pi}{4}$
3. $\tan\frac{\pi}{3}$
4. $\sin\frac{\pi}{2}$
5. $\tan\pi$
6. $\cos\frac{2\pi}{3}$
7. $\cos 0$
8. $\tan\frac{3\pi}{2}$
9. $\sin\frac{5\pi}{4}$
10. $\sin\frac{5\pi}{3}$
11. $\tan\frac{11\pi}{6}$
12. $\sin 0$

13. $\cos\frac{7\pi}{4}$

14. $\cos(\frac{-3\pi}{2})$

15. $\tan(\frac{-3\pi}{4})$

16. $\sin\frac{17\pi}{6}$

17. $\cos\frac{11\pi}{3}$

18. $\tan 100\pi$

19. $\sin(-\frac{\pi}{6})$

20. $\cos(\frac{-2\pi}{3})$

21. $\tan(\frac{-9\pi}{2})$

Trigonometric Identities: YOU NEED TO MEMORIZE THESE!

- **Reciprocal identities**

$$\sin u = \frac{1}{\csc u} \quad \cos u = \frac{1}{\sec u}$$
$$\tan u = \frac{1}{\cot u} \quad \cot u = \frac{1}{\tan u}$$
$$\csc u = \frac{1}{\sin u} \quad \sec u = \frac{1}{\cos u}$$

- **Pythagorean Identities**

$$\sin^2 u + \cos^2 u = 1$$
$$1 + \tan^2 u = \sec^2 u$$
$$1 + \cot^2 u = \csc^2 u$$

- **Quotient Identities**

$$\tan u = \frac{\sin u}{\cos u} \quad \cot u = \frac{\cos u}{\sin u}$$

- **Co-Function Identities**

$$\sin(\frac{\pi}{2} - u) = \cos u \quad \cos(\frac{\pi}{2} - u) = \sin u$$
$$\tan(\frac{\pi}{2} - u) = \cot u \quad \cot(\frac{\pi}{2} - u) = \tan u$$
$$\csc(\frac{\pi}{2} - u) = \sec u \quad \sec(\frac{\pi}{2} - u) = \csc u$$

- **Parity Identities (Even & Odd)**

$$\sin(-u) = -\sin u \quad \cos(-u) = \cos u$$
$$\tan(-u) = -\tan u \quad \cot(-u) = -\cot u$$
$$\csc(-u) = -\csc u \quad \sec(-u) = \sec u$$

Verify the following trigonometric identities.

1. $\cos x + \sin x \tan x = \sec x$

2. $\dfrac{\csc x - \sin x}{\sin x \csc x} = \csc x - \sin x$

3. $\dfrac{1}{\tan \beta} + \tan \beta = \dfrac{\sec^2 \beta}{\tan \beta}$

4. $\dfrac{1 + \sin \theta}{\cos \theta} + \dfrac{\cos \theta}{1 + \sin \theta} = 2 \sec \theta$

5. $\sec y + \tan y = \dfrac{\cos y}{1 - \sin y}$

Solutions to Exercises

1. LHS $\rightarrow$ $\cos x+\sin x\tan x$

$$
\begin{aligned}
&= \cos x+\sin x\left(\frac{\sin x}{\cos x}\right)\\
&= \cos x+\frac{\sin^2 x}{\cos x}\\
&= \frac{\cos^2 x}{\cos x}+\frac{\sin^2 x}{\cos x}\\
&= \frac{\cos^2 x+\sin^2 x}{\cos x}\\
&= \frac{1}{\cos x}\\
&= \sec x
\end{aligned}
$$

2. LHS $\rightarrow$ $\frac{\csc x-\sin x}{\sin x\csc x}$

$$
\begin{aligned}
&= \frac{1}{\sin x\csc x}(\csc x-\sin x)\\
&= \frac{1}{\sin x\csc x}\csc x-\frac{1}{\sin x\csc x}\sin x\\
&= \frac{1}{\sin x}-\frac{1}{\csc x}\\
&= \csc x-\sin x
\end{aligned}
$$

3. LHS $\rightarrow$ $\frac{1}{\tan\beta}+\tan\beta$

$$
\begin{aligned}
&= \frac{1}{\tan\beta}+\frac{\tan^2\beta}{\tan\beta}\\
&= \frac{1+\tan^2\beta}{\tan\beta}\\
&= \frac{\sec^2\beta}{\tan\beta}
\end{aligned}
$$

4. LHS $\rightarrow$ $\frac{1+\sin\theta}{\cos\theta}+\frac{\cos\theta}{1+\sin\theta}$

$$
\begin{aligned}
&= \frac{1+\sin\theta}{\cos\theta}\left(\frac{1+\sin\theta}{1+\sin\theta}\right)+\frac{\cos\theta}{1+\sin\theta}\left(\frac{\cos\theta}{\cos\theta}\right)\\
&= \frac{(1+\sin\theta)^2+\cos^2\theta}{\cos\theta(1+\sin\theta)}\\
&= \frac{1+2\sin\theta+\sin^2\theta+\cos^2\theta}{\cos\theta(1+\sin\theta)}\\
&= \frac{1+2\sin\theta+1}{\cos\theta(1+\sin\theta)}\\
&= \frac{2+2\sin\theta}{\cos\theta(1+\sin\theta)}\\
&= \frac{2(1+\sin\theta)}{\cos\theta(1+\sin\theta)}\\
&= \frac{2}{\cos\theta}\\
&= 2\sec\theta
\end{aligned}
$$

5. RHS $\rightarrow$ $\frac{\cos y}{1-\sin y}$

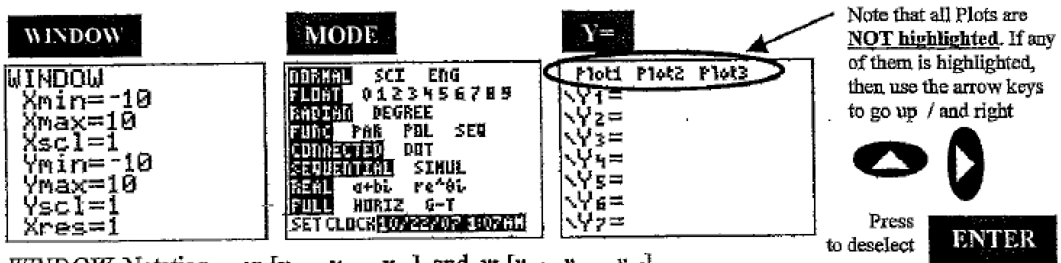
$$
\begin{aligned}
&= \frac{\cos y}{1-\sin y}\left(\frac{1+\sin y}{1+\sin y}\right)\\
&= \frac{\cos y(1+\sin y)}{1-\sin^2 y}\\
&= \frac{\cos y(1+\sin y)}{\cos^2 y}\\
&= \frac{1+\sin y}{\cos y}\\
&= \frac{1}{\cos y}+\frac{\sin y}{\cos y}\\
&= \sec y+\tan y
\end{aligned}
$$

You will also need to buy a graphing calculator for this course. The department recommends the TI-83 or TI-84. You will be receiving special Calculus programs from your teacher. If you have these types of calculators, they will easily link to your teacher's calculator to input the programs.

Below are instructions to follow to learn how to use the calculator. After you have followed the instructions given and understand the basic keys of your graphing calculator, there are 6 problems that need to be completed. Please check your answers.

TI-83/84 Plus Graphing Calculator Worksheet #2

The graphing calculator is set in the following WINDOW, MODE, and Y=, settings. Resetting your calculator brings it back to these original settings.

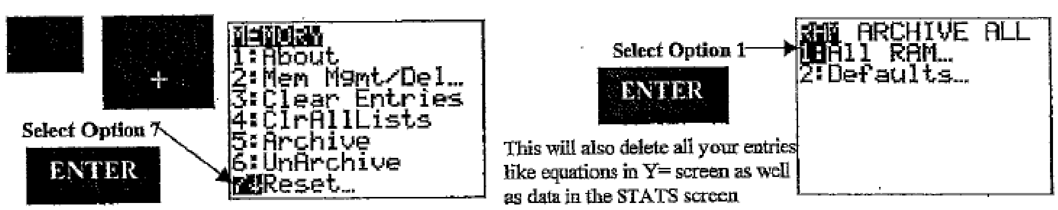


WINDOW Notation x: $[x_{min}, x_{max}, x_{scl}]$ and y: $[y_{min}, y_{max}, y_{scl}]$

Original Setting x: $[-10, 10, 1]$ and y: $[-10, 10, 1]$

Resetting Calculator to Factory Setting:

- when the user have used the calculator in various ways and it is difficult to go back to the original setting.
- when the user lend the calculator to others and they have messed up the original setting.
- this should be done before a test or after you lend the calculator to a friend



Adjusting WINDOW of a graph:
Sometimes, a graph needs to be set with a customize WINDOW. This is similar to setting the intervals and the ranges for both x- and y- axis.

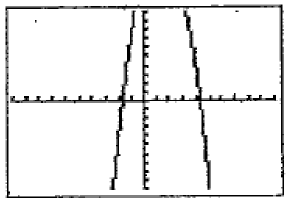
Example 1: Graph $y = -2x^2 + 5x + 15$.

Y= To enter negative sign, press (–)

```
Plot1 Plot2 Plot3
\Y1=-2X²+5X+15
\Y2=
\Y3=
\Y4=
\Y5=
\Y6=
\Y7=
```

To enter X, press **X,T,θ,n**

GRAPH

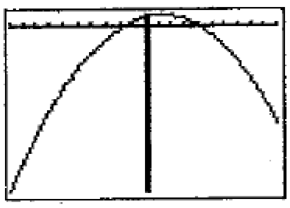


ZOOM

```
ZOOM MEMORY
4↑ZDecimal
5:ZSquare
6:ZStandard
7:ZTrig
8:ZInteger
9:ZoomStat
0:ZoomFit
```

Scroll down with ▼ **and press ENTER or Select Option 0**

Note: We use the subtraction button **–** between terms. Otherwise, we use (–) for negative signs.



```
WINDOW
 Xmin=-10
 Xmax=10
 Xscl=1
 Ymin=-235
 Ymax=18.123585…
 Yscl=1
 Xres=1
```

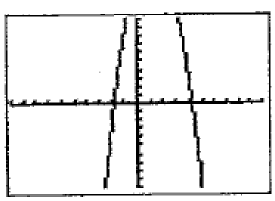
The ZoomFit option does not give a neat WINDOW setting, but it allows us to see the whole graph

To quickly reset the original WINDOW setting without resetting the entire calculator:

ZOOM

```
ZOOM MEMORY
1:ZBox
2:Zoom In
3:Zoom Out
4:ZDecimal
5:ZSquare
6:ZStandard
7↓ZTrig
```

Scroll down with ▼ **and press ENTER or Select Option 6**



WINDOW

```
WINDOW
 Xmin=-10
 Xmax=10
 Xscl=1
 Ymin=-10
 Ymax=10
 Yscl=1
 Xres=1
```

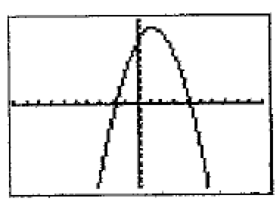
Note the WINDOW goes back to the original setting.

Now, we try using a customize WINDOW setting to x: [–10, 10, 1] and y: [–20, 20, 1].

WINDOW

```
WINDOW
 Xmin=-10
 Xmax=10
 Xscl=1
 Ymin=-20
 Ymax=20
 Yscl=1
 Xres=1
```

GRAPH

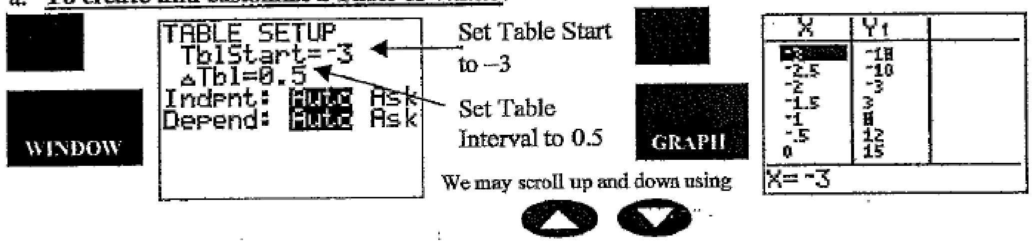


Note that now the graph fits nicely.

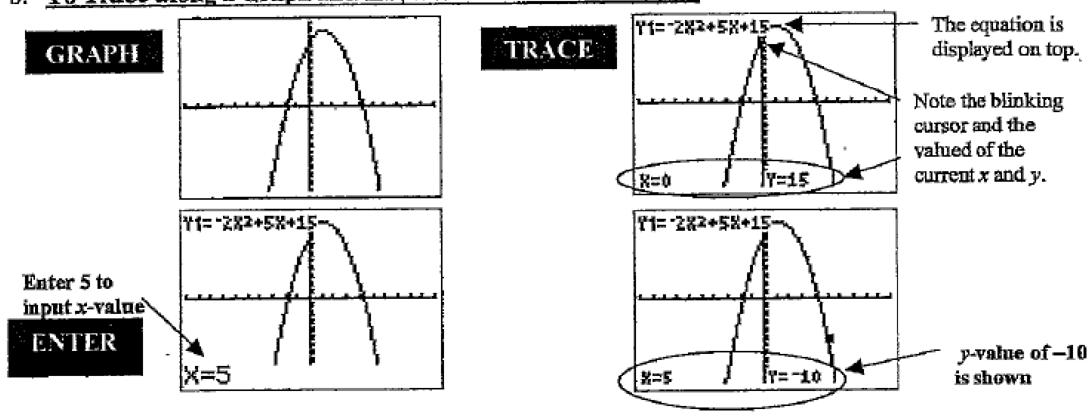
Example 2: Using the graph $y = -2x^2 + 5x + 15$ from the previous example,

a. Create a table of values starting at $x = -3$ with an increasing interval of 0.5.
b. Trace the graph and find the value of y when $x = 5$ from the graph.
c. What is the y-intercept of this graph?
d. Determine the x-intercepts.
e. Give the coordinates of where the maximum value of this graph occurs.
f. Solve $-2x^2 + 5x + 15 > 0$ and then solve $-2x^2 + 5x + 15 \le 0$.

a. **To create and customize a Table of Values:**

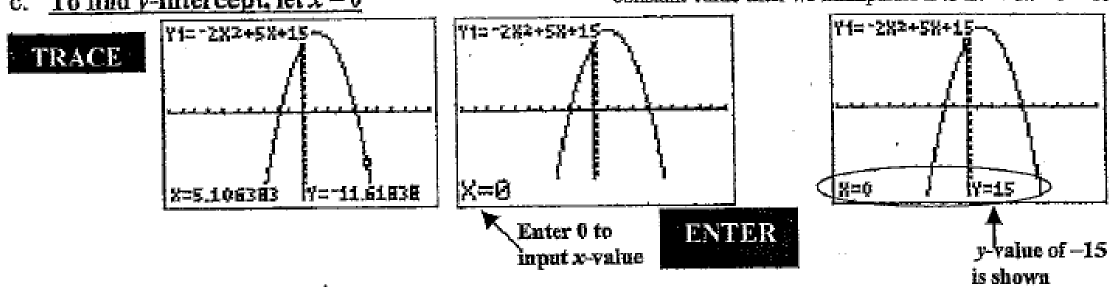


b. **To Trace along a Graph and find a Y-value from an X-value:**

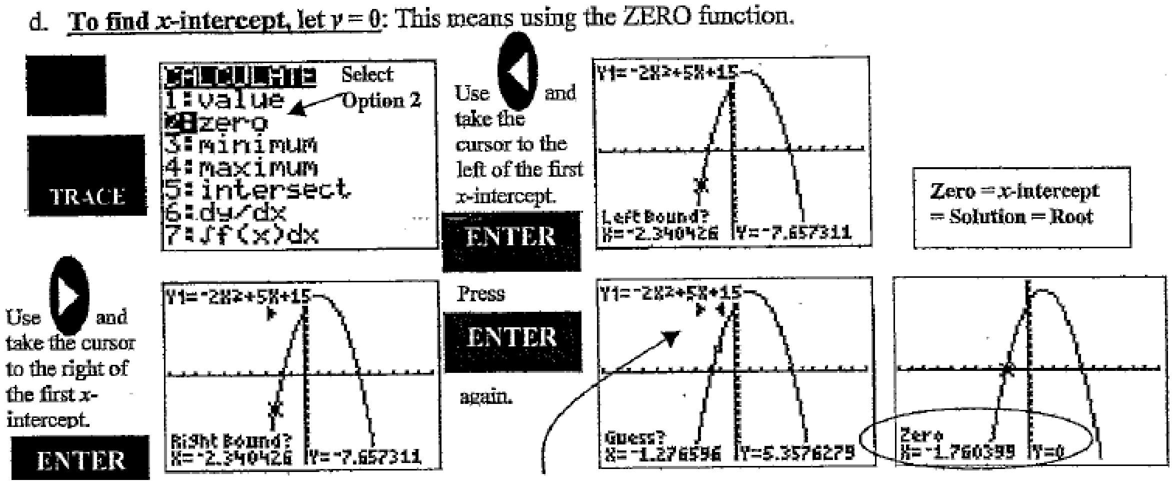


c. **To find y-intercept, let $x = 0$**

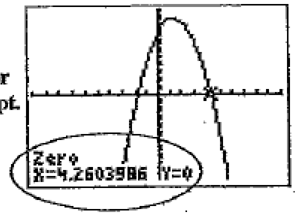
Note the y-intercept of a quadratic equation is its constant value after we manipulate it to $ax^2 + bx + c = 0$.



d. **To find x-intercept, let $y = 0$**: This means using the ZERO function.



Do the same steps for the second x-intercept.

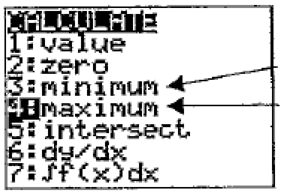


Note the two little triangles that appear. They indicate the calculator will find the x-intercept within that range.

Because the original quadratic equation, $y = -2x^2 + 5x + 15$, is not factorable, these **solutions are the decimal equivalents of the roots found from the quadratic formula**. However, ***we prefer the exact values*** from the quadratic formula to their decimal equivalents.

e. **To find the coordinates of the Maximum (or the Minimum) of a Graph:**

TRACE

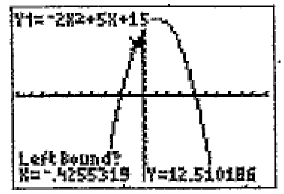


Select Option 3 for Minimum

Select Option 4 for Maximum

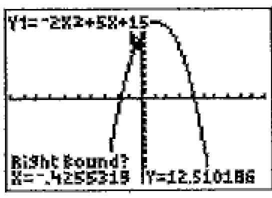
Use [left arrow] and take the cursor to the left of the Maximum point

ENTER



Use [right arrow] and take the cursor to the right of the Maximum point.

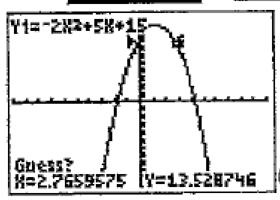
ENTER

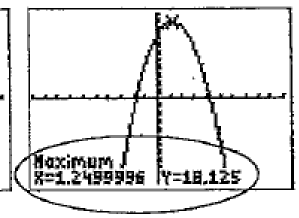


Press

ENTER

again.

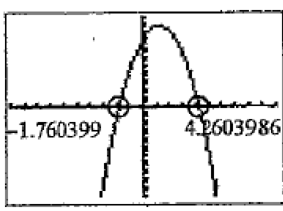




f. **Solve Inequalities from Graphing:** $(-2x^2 + 5x + 15 > 0)$ and $(-2x^2 + 5x + 15 \le 0)$

GRAPH

x: [−10, 10, 1]
and
y: [−20, 20, 1]



when $y > 0$ (positive y-values)

when $y = 0$ (all y-values of x-axis = 0)

when $y > 0$ (negative y-values)

$$x\text{-intercepts} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(5) \pm \sqrt{(5)^2 - 4(-2)(15)}}{2(-2)} = \frac{-5 \pm \sqrt{145}}{-4} = \frac{5 \pm \sqrt{145}}{4}$$

$$x = \frac{5 - \sqrt{145}}{4} \approx -1.760399 \qquad x = \frac{5 + \sqrt{145}}{4} \approx 4.2603986$$

For $-2x^2 + 5x + 15 > 0$, it is the same as when $y > 0$.

Approx Solution: $-1.760399 < x < 4.2603986$

Exact Solution: $\frac{5 - \sqrt{145}}{4} < x < \frac{5 + \sqrt{145}}{4}$

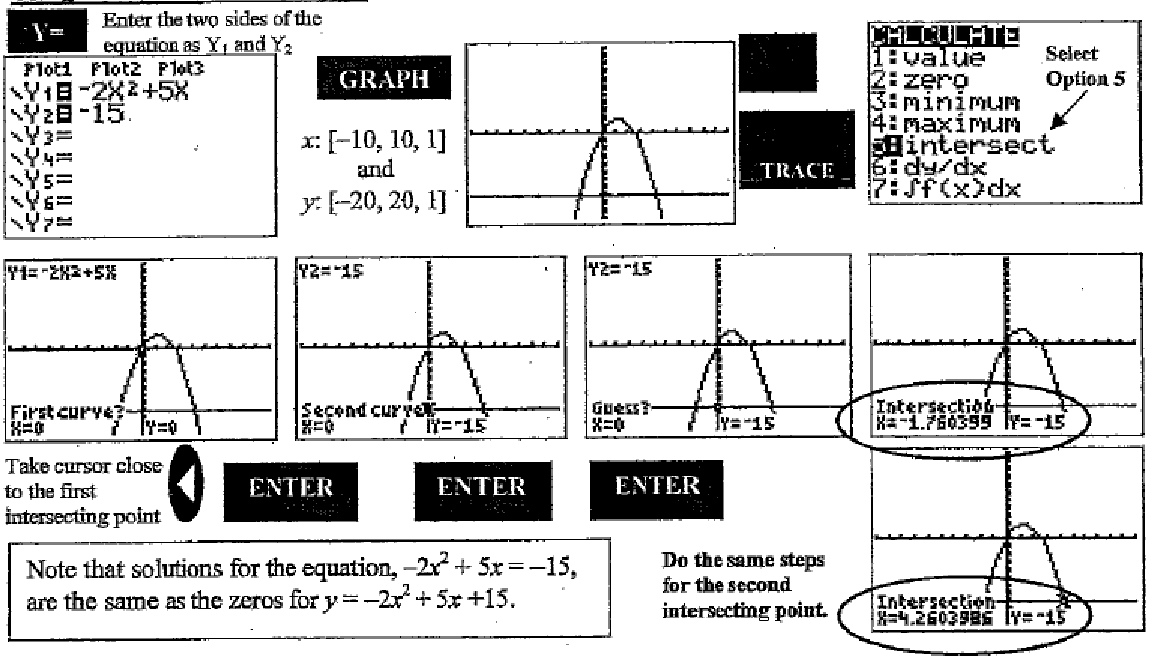
For $-2x^2 + 5x + 15 \le 0$, it is the same as when $y \le 0$.

Approx Solution: $x \le -1.760399$ or $x \ge 4.2603986$

Exact Solution: $x \le \frac{5 - \sqrt{145}}{4}$ or $x \ge \frac{5 + \sqrt{145}}{4}$

Example 3: Solve $-2x^2 + 5x = -15$ using the INTERSECT function.

Using the INTERSECT function:



Note that solutions for the equation, $-2x^2 + 5x = -15$, are the same as the zeros for $y = -2x^2 + 5x + 15$.

Now try these six problems:

1) Given: f(x) = x^6 - 3x^4 + 2x + 1
   Find all roots to the nearest 0.001

2) Given: f(x) = -2 cos 4x + 2x + 1 from [-2pi, 2pi]
   Find all roots to the nearest 0.001. (Note: All trig functions are done in radian mode.)

3) Given: f(x) = 10x^3 + 220x^2 + 113x - 3
   Find all roots to the nearest 0.001

4) Given: f(x) = 2x^3 + 3x^2- 8x - 1 and g(x) = x^2 + 1
   Find the point of intersection.

5) How many times does the graph of $y = 0.4x$ intersect the graph of $y = \cos(3x)$?

6) Given: f(x) = 2x^4 + 3x^3 - x^2 + 2x - 7

a) Determine the x- and y-coordinates of the min. value on the graph.

b) Size the x-window from [-10,10]. Find the maximum and minimum values of f(x) over the interval $-10 < x < 10$.

Check your Answers:

1. x = -1.802, -0.445, 1.247, 1.466

2. x = -1.037, -0.357, 0.199

3. x = -21.473, -0.552, 0.025

4. (-2.461, 7.054) (1.7, 3.889) (-0.239, 1.057)

5. 5 times

6. a) (-1.424, -12.315)
   b) none not closed on the interval