

## Summer Work for AP Calculus AB

1. Go to [Larsonprecalculus.com](http://Larsonprecalculus.com) and watch the videos for sections 9.1-9.2 on Sequences and Series, then do:  
p. 676: 1-55 odd
2. Go to [Larsonprecalculus.com](http://Larsonprecalculus.com) and watch the videos for 10.1-10.4 and 10.6-10.8 on Conics, Parametric Equations and Polar Coordinates, then do:  
p. 764: 1-43 odd, 53-103 odd
3. Go to [Larsoncalculus.com](http://Larsoncalculus.com) and watch the videos for 7.3 on the Shell Method then do:  
p. 462: 1-35 odd

## Review Exercises

See CalcChat.com for tutorial help and worked-out solutions to odd-numbered exercises.

**9.1** Writing the Terms of a Sequence In Exercises 1–4, write the first five terms of the sequence. (Assume that  $n$  begins with 1.)

$$1. a_n = 2 + \frac{6}{n} \qquad 2. a_n = \frac{(-1)^n 5n}{2n - 1}$$

$$3. a_n = \frac{72}{n!} \qquad 4. a_n = n(n - 1)$$

Finding the  $n$ th Term of a Sequence In Exercises 5–8, write an expression for the apparent  $n$ th term ( $a_n$ ) of the sequence. (Assume that  $n$  begins with 1.)

$$5. -2, 2, -2, 2, -2, \dots \qquad 6. -1, 2, 7, 14, 23, \dots$$

$$7. 4, 2, \frac{4}{3}, 1, \frac{2}{3}, \dots \qquad 8. 1, -\frac{1}{2}, \frac{1}{3}, -\frac{1}{4}, \frac{1}{5}, \dots$$

Simplifying a Factorial Expression In Exercises 9–12, simplify the factorial expression.

$$9. 9! \qquad 10. 4! \cdot 0!$$

$$11. \frac{3! \cdot 5!}{6!} \qquad 12. \frac{7! \cdot 6!}{6! \cdot 8!}$$

Finding a Sum In Exercises 13 and 14, find the sum.

$$13. \sum_{j=1}^4 \frac{6}{j^2} \qquad 14. \sum_{k=1}^{10} 2k^3$$

Using Sigma Notation to Write a Sum In Exercises 15 and 16, use sigma notation to write the sum.

$$15. \frac{1}{2(1)} + \frac{1}{2(2)} + \frac{1}{2(3)} + \dots + \frac{1}{2(20)}$$

$$16. \frac{1}{2} + \frac{2}{3} + \frac{3}{4} + \dots + \frac{9}{10}$$

Finding the Sum of an Infinite Series In Exercises 17 and 18, find the sum of the infinite series.

$$17. \sum_{i=1}^{\infty} \frac{4}{10^i} \qquad 18. \sum_{k=1}^{\infty} 2 \left( \frac{1}{100} \right)^k$$

**19. Compound Interest** An investor deposits \$10,000 in an account that earns 2.25% interest compounded monthly. The balance in the account after  $n$  months is given by

$$A_n = 10,000 \left( 1 + \frac{0.0225}{12} \right)^n, \quad n = 1, 2, 3, \dots$$

- (a) Write the first 10 terms of the sequence.  
 (b) Find the balance in the account after 10 years by computing the 120th term of the sequence.

**20. Lottery Ticket Sales** The total sales  $a_n$  (in billions of dollars) of lottery tickets in the United States from 2001 through 2010 can be approximated by the model

$$a_n = -0.18n^2 + 3.7n + 35, \quad n = 1, 2, \dots, 10$$

where  $n$  is the year, with  $n = 1$  corresponding to 2001. Write the terms of this finite sequence. Use a graphing utility to construct a bar graph that represents the sequence. (Source: TLF Publications, Inc.)

**9.2** Determining Whether a Sequence Is Arithmetic In Exercises 21–24, determine whether the sequence is arithmetic. If so, then find the common difference.

$$21. 6, -1, -8, -15, -22, \dots$$

$$22. 0, 1, 3, 6, 10, \dots \qquad 23. \frac{1}{2}, 1, \frac{3}{2}, 2, \frac{5}{2}, \dots$$

$$24. 1, \frac{15}{16}, \frac{7}{8}, \frac{13}{16}, \frac{3}{4}, \dots$$

Finding the  $n$ th Term In Exercises 25–28, find a formula for  $a_n$  for the arithmetic sequence.

$$25. a_1 = 7, d = 12 \qquad 26. a_1 = 28, d = -5$$

$$27. a_2 = 93, a_6 = 65 \qquad 28. a_7 = 8, a_{13} = 6$$

Writing the Terms of an Arithmetic Sequence In Exercises 29 and 30, write the first five terms of the arithmetic sequence.

$$29. a_1 = 3, d = 11 \qquad 30. a_1 = 25, a_{k+1} = a_k + 3$$

**31. Sum of a Finite Arithmetic Sequence** Find the sum of the first 100 positive multiples of 7.

**32. Sum of a Finite Arithmetic Sequence** Find the sum of the integers from 40 to 90 (inclusive).

Finding a Partial Sum In Exercises 33–36, find the partial sum.

$$33. \sum_{j=1}^{10} (2j - 3) \qquad 34. \sum_{j=1}^8 (20 - 3j)$$

$$35. \sum_{k=1}^{11} \left( \frac{2}{3}k + 4 \right) \qquad 36. \sum_{k=1}^{25} \left( \frac{3k + 1}{4} \right)$$

**37. Job Offer** The starting salary for a job is \$43,800 with a guaranteed increase of \$1950 per year. Determine (a) the salary during the fifth year and (b) the total compensation through five full years of employment.

**38. Baling Hay** In the first two trips baling hay around a large field, a farmer obtains 123 bales and 112 bales, respectively. Because each round gets shorter, the farmer estimates that the same pattern will continue. Estimate the total number of bales made after the farmer takes another six trips around the field.

**9.3 Determining Whether a Sequence Is Geometric** In Exercises 39–42, determine whether the sequence is geometric. If so, then find the common ratio.

39. 6, 12, 24, 48, . . .

40. 54, -18, 6, -2, . . .

41.  $\frac{1}{5}, -\frac{3}{5}, \frac{9}{5}, -\frac{27}{5}, \dots$

42.  $\frac{1}{4}, \frac{2}{5}, \frac{3}{6}, \frac{4}{7}, \dots$

**Writing the Terms of a Geometric Sequence** In Exercises 43–46, write the first five terms of the geometric sequence.

43.  $a_1 = 2, r = 15$

44.  $a_1 = 4, r = -\frac{1}{4}$

45.  $a_1 = 9, a_3 = 4$

46.  $a_1 = 2, a_3 = 12$

**Finding a Term of a Geometric Sequence** In Exercises 47–50, write an expression for the  $n$ th term of the geometric sequence. Then find the 10th term of the sequence.

47.  $a_1 = 18, a_2 = -9$

48.  $a_3 = 6, a_4 = 1$

49.  $a_1 = 100, r = 1.05$

50.  $a_1 = 5, r = 0.2$

**Sum of a Finite Geometric Sequence** In Exercises 51–56, find the sum of the finite geometric sequence.

51.  $\sum_{i=1}^7 2^{i-1}$


52.  $\sum_{i=1}^5 3^{i-1}$

53.  $\sum_{i=1}^4 \left(\frac{1}{5}\right)^i$

54.  $\sum_{i=1}^6 \left(\frac{1}{3}\right)^{i-1}$

55.  $\sum_{i=1}^5 (2)^{i-1}$

56.  $\sum_{i=1}^4 6(3)^i$

 **Sum of a Finite Geometric Sequence** In Exercises 57 and 58, use a graphing utility to find the sum of the finite geometric sequence.

57.  $\sum_{i=1}^{10} 10\left(\frac{3}{5}\right)^{i-1}$

58.  $\sum_{i=1}^{15} 20(0.2)^{i-1}$

**Sum of an Infinite Geometric Sequence** In Exercises 59–62, find the sum of the infinite geometric series.

59.  $\sum_{i=0}^{\infty} \left(\frac{2}{8}\right)^i$

60.  $\sum_{i=0}^{\infty} (0.5)^i$

61.  $\sum_{k=1}^{\infty} 4\left(\frac{2}{3}\right)^{k-1}$

62.  $\sum_{k=1}^{\infty} 1.3\left(\frac{1}{10}\right)^{k-1}$

63. **Depreciation** A paper manufacturer buys a machine for \$120,000. During the next 5 years, it will depreciate at a rate of 30% per year. (In other words, at the end of each year the depreciated value will be 70% of what it was at the beginning of the year.)

(a) Find the formula for the  $n$ th term of a geometric sequence that gives the value of the machine  $t$  full years after it was purchased.

(b) Find the depreciated value of the machine after 5 full years.

64. **Annuity** An investor deposits \$800 in an account on the first day of each month for 10 years. The account pays 3%, compounded monthly. What will the balance be at the end of 10 years?

**9.4 Using Mathematical Induction** In Exercises 65–68, use mathematical induction to prove the formula for every positive integer  $n$ .

65.  $3 + 5 + 7 + \dots + (2n + 1) = n(n + 2)$

66.  $1 + \frac{3}{2} + 2 + \frac{5}{2} + \dots + \frac{1}{2}(n + 1) = \frac{n}{4}(n + 3)$

67.  $\sum_{i=0}^{n-1} ar^i = \frac{a(1 - r^n)}{1 - r}$

68.  $\sum_{i=0}^{n-1} (a + kd) = \frac{n}{2}[2a + (n - 1)d]$

**Finding a Formula for a Sum** In Exercises 69–72, use mathematical induction to find a formula for the sum of the first  $n$  terms of the sequence.

69. 9, 13, 17, 21, . . .

70. 68, 60, 52, 44, . . .

71.  $1, \frac{3}{5}, \frac{9}{25}, \frac{27}{125}, \dots$

72.  $12, -1, \frac{1}{12}, -\frac{1}{144}, \dots$

**Finding a Sum** In Exercises 73 and 74, find the sum using the formulas for the sums of powers of integers.

73.  $\sum_{n=1}^{78} n$

74.  $\sum_{n=1}^6 (n^5 - n^3)$

**Linear Model, Quadratic Model, or Neither?** In Exercises 75 and 76, write the first five terms of the sequence beginning with the given term. Then calculate the first and second differences of the sequence. State whether the sequence has a perfect linear model, a perfect quadratic model, or neither.

75.  $a_1 = 5$

76.  $a_1 = -3$

$a_n = a_{n-1} + 5$

$a_n = a_{n-1} - 2n$

**9.5 Finding a Binomial Coefficient** In Exercises 77 and 78, find the binomial coefficient.

77.  ${}_6C_4$

78.  ${}_{12}C_3$

**Using Pascal's Triangle** In Exercises 79 and 80, evaluate using Pascal's Triangle.

79.  $\binom{7}{2}$

80.  $\binom{10}{4}$

**Expanding an Expression** In Exercises 81–84, use the Binomial Theorem to expand and simplify the expression. (Remember that  $i = \sqrt{-1}$ .)

81.  $(x + 4)^4$

82.  $(a - 3b)^5$

83.  $(5 + 2i)^4$

84.  $(4 - 5i)^3$

- 1.** 8, 5, 4,  $\frac{7}{2}$ ,  $\frac{16}{5}$     **3.** 72, 36, 12, 3,  $\frac{3}{5}$     **5.**  $a_n = 2(-1)^n$   
**7.**  $a_n = \frac{4}{n}$     **9.** 362,880    **11.** 1    **13.**  $\frac{205}{24}$   
**15.**  $\sum_{k=1}^{20} \frac{1}{2k}$     **17.**  $\frac{4}{9}$   
**19.** (a)  $A_1 = \$10,018.75$   
 $A_2 \approx \$10,037.54$   
 $A_3 \approx \$10,056.36$   
 $A_4 \approx \$10,075.21$   
 $A_5 \approx \$10,094.10$   
 $A_6 \approx \$10,113.03$   
 $A_7 \approx \$10,131.99$   
 $A_8 \approx \$10,150.99$   
 $A_9 \approx \$10,170.02$   
 $A_{10} \approx \$10,189.09$   
(b) \$12,520.59  
**21.** Arithmetic sequence,  $d = -7$   
**23.** Arithmetic sequence,  $d = \frac{1}{2}$     **25.**  $a_n = 12n - 5$   
**27.**  $a_n = -7n + 107$     **29.** 3, 14, 25, 36, 47    **31.** 35,350  
**33.** 80    **35.** 88    **37.** (a) \$51,600    (b) \$238,500  
**39.** Geometric sequence,  $r = 2$   
**41.** Geometric sequence,  $r = -3$   
**43.** 2, 30, 450, 6750, 101,250  
**45.** 9, 6, 4,  $\frac{8}{3}$ ,  $\frac{16}{9}$  or 9, -6, 4,  $-\frac{8}{3}$ ,  $\frac{16}{9}$     **47.**  $a_n = 18\left(-\frac{1}{2}\right)^{n-1}; \frac{9}{512}$   
**49.**  $a_n = 100(1.05)^{n-1}$ ; About 155.133    **51.** 127    **53.**  $\frac{15}{16}$   
**55.** 31    **57.** 24.85    **59.** 8    **61.** 12

## Review Exercises

See CalcChat.COM for tutorial help and worked-out solutions to odd-numbered exercises.

**10.1** Finding the Inclination of a Line In Exercises 1–4, find the inclination  $\theta$  (in radians and degrees) of the line with the given characteristics.

1. Passes through the points  $(-1, 2)$  and  $(2, 5)$
2. Passes through the points  $(3, 4)$  and  $(-2, 7)$
3. Equation:  $y = 2x + 4$
4. Equation:  $x - 5y = 7$

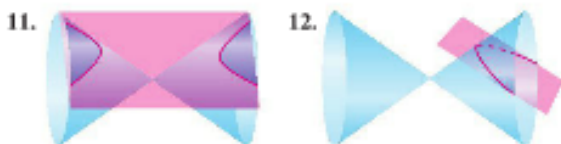
Finding the Angle Between Two Lines In Exercises 5–8, find the angle  $\theta$  (in radians and degrees) between the lines.

- |                        |                           |
|------------------------|---------------------------|
| 5. $4x + y = 2$        | 6. $-5x + 3y = 3$         |
| $-5x + y = -1$         | $-2x + 3y = 1$            |
| 7. $2x - 7y = 8$       | 8. $0.02x + 0.07y = 0.18$ |
| $\frac{2}{5}x + y = 0$ | $0.09x - 0.04y = 0.17$    |

Finding the Distance Between a Point and a Line In Exercises 9 and 10, find the distance between the point and the line.

- | Point        | Line         |
|--------------|--------------|
| 9. $(5, 3)$  | $x - y = 10$ |
| 10. $(0, 4)$ | $x + 2y = 2$ |

**10.2** Forming a Conic Section In Exercises 11 and 12, state what type of conic is formed by the intersection of the plane and the double-napped cone.



Finding the Standard Equation of a Parabola In Exercises 13–16, find the standard form of the equation of the parabola with the given characteristics. Then sketch the parabola.

- |   |  |
|---|--|
| 13. Vertex: $(0, 0)$<br>Focus: $(4, 0)$     | 14. Vertex: $(2, 0)$<br>Focus: $(0, 0)$      |
| 15. Vertex: $(0, 2)$<br>Directrix: $x = -3$ | 16. Vertex: $(-3, -3)$<br>Directrix: $y = 0$ |

Finding the Tangent Line at a Point on a Parabola In Exercises 17 and 18, find the equation of the tangent line to the parabola at the given point.

17.  $y = 2x^2$ ,  $(-1, 2)$
18.  $x^2 = -2y$ ,  $(-4, -8)$

19. **Architecture** A parabolic archway is 10 meters high at the vertex. At a height of 8 meters, the width of the archway is 6 meters (see figure). How wide is the archway at ground level?

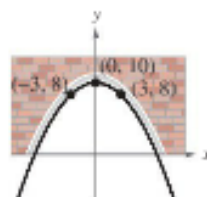


Figure for 19

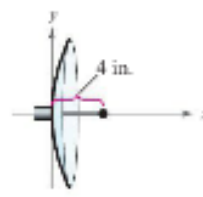


Figure for 20

20. **Parabolic Microphone** The receiver of a parabolic microphone is at the focus of its parabolic reflector, 4 inches from the vertex (see figure). Write an equation of a cross section of the reflector with its focus on the positive  $x$ -axis and its vertex at the origin.

**10.3** Finding the Standard Equation of an Ellipse In Exercises 21–24, find the standard form of the equation of the ellipse with the given characteristics.

21. Vertices:  $(-2, 0)$ ,  $(8, 0)$ ; foci:  $(0, 0)$ ,  $(6, 0)$
22. Vertices:  $(4, 3)$ ,  $(4, 7)$ ; foci:  $(4, 4)$ ,  $(4, 6)$
23. Vertices:  $(0, 1)$ ,  $(4, 1)$ ; endpoints of the minor axis:  $(2, 0)$ ,  $(2, 2)$
24. Vertices:  $(-4, -1)$ ,  $(-4, 11)$ ; endpoints of the minor axis:  $(-6, 5)$ ,  $(-2, 5)$
25. **Architecture** A contractor plans to construct a semielliptical arch 10 feet wide and 4 feet high. Where should the foci be placed in order to sketch the arch?
26. **Wading Pool** You are building a wading pool that is in the shape of an ellipse. Your plans give an equation for the elliptical shape of the pool measured in feet as

$$\frac{x^2}{324} + \frac{y^2}{196} = 1.$$

Find the longest distance across the pool, the shortest distance, and the distance between the foci.

Sketching an Ellipse In Exercises 27–30, find the center, vertices, foci, and eccentricity of the ellipse. Then sketch the ellipse.

27.  $\frac{(x+1)^2}{25} + \frac{(y-2)^2}{49} = 1$
28.  $\frac{(x-5)^2}{1} + \frac{(y+3)^2}{36} = 1$
29.  $16x^2 + 9y^2 - 32x + 72y + 16 = 0$
30.  $4x^2 + 25y^2 + 16x - 150y + 141 = 0$



**Polar-to-Rectangular Conversion** In Exercises 71–74, a point in polar coordinates is given. Convert the point to rectangular coordinates.

$$71. \left(-1, \frac{\pi}{3}\right) \qquad 72. \left(2, \frac{5\pi}{4}\right)$$

$$73. \left(3, \frac{3\pi}{4}\right) \qquad 74. \left(0, \frac{\pi}{2}\right)$$

**Rectangular-to-Polar Conversion** In Exercises 75–78, a point in rectangular coordinates is given. Convert the point to polar coordinates.

$$75. (0, 1) \qquad 76. (-\sqrt{5}, \sqrt{5})$$

$$77. (4, 6) \qquad 78. (3, -4)$$

**Converting a Rectangular Equation to Polar Form** In Exercises 79–84, convert the rectangular equation to polar form.

$$79. x^2 + y^2 = 81 \qquad 80. x^2 + y^2 = 48$$

$$81. x^2 + y^2 - 6y = 0 \qquad 82. x^2 + y^2 - 4x = 0$$

$$83. xy = 5 \qquad 84. xy = -2$$

**Converting a Polar Equation to Rectangular Form** In Exercises 85–90, convert the polar equation to rectangular form.

$$85. r = 5 \qquad 86. r = 12$$

$$87. r = 3 \cos \theta \qquad 88. r = 8 \sin \theta$$

$$89. r^2 = \sin \theta \qquad 90. r^2 = 4 \cos 2\theta$$

**10.8 Sketching the Graph of a Polar Equation** In Exercises 91–100, sketch the graph of the polar equation using symmetry, zeros, maximum  $r$ -values, and any other additional points.

$$91. r = 6 \qquad 92. r = 11$$

$$93. r = 4 \sin 2\theta \qquad 94. r = \cos 5\theta$$

$$95. r = -2(1 + \cos \theta) \qquad 96. r = 1 - 4 \cos \theta$$

$$97. r = 2 + 6 \sin \theta \qquad 98. r = 5 - 5 \cos \theta$$

$$99. r = -3 \cos 2\theta \qquad 100. r^2 = \cos 2\theta$$

**Identifying Types of Polar Graphs** In Exercises 101–104, identify the type of polar graph and use a graphing utility to graph the equation.

$$101. r = 3(2 - \cos \theta) \qquad 102. r = 5(1 - 2 \cos \theta)$$

$$103. r = 8 \cos 3\theta \qquad 104. r^2 = 2 \sin 2\theta$$

**10.9 Sketching a Conic** In Exercises 105–108, identify the conic and sketch its graph.

$$105. r = \frac{1}{1 + 2 \sin \theta} \qquad 106. r = \frac{6}{1 + \sin \theta}$$

$$107. r = \frac{4}{5 - 3 \cos \theta} \qquad 108. r = \frac{16}{4 + 5 \cos \theta}$$

**Finding the Polar Equation of a Conic** In Exercises 109–112, find a polar equation of the conic with its focus at the pole.

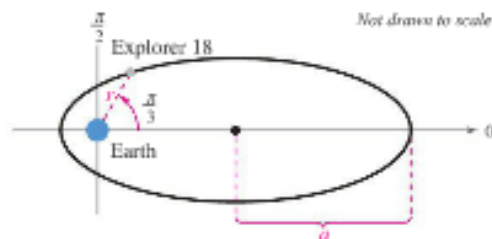
$$109. \text{Parabola} \qquad \text{Vertex: } (2, \pi)$$

$$110. \text{Parabola} \qquad \text{Vertex: } (2, \pi/2)$$

$$111. \text{Ellipse} \qquad \text{Vertices: } (5, 0), (1, \pi)$$

$$112. \text{Hyperbola} \qquad \text{Vertices: } (1, 0), (7, 0)$$

113. **Explorer 18** On November 27, 1963, the United States launched Explorer 18. Its low and high points above the surface of Earth were 110 miles and 122,800 miles, respectively. The center of Earth was at one focus of the orbit (see figure). Find the polar equation of the orbit and find the distance between the surface of Earth (assume Earth has a radius of 4000 miles) and the satellite when  $\theta = \pi/3$ .



114. **Asteroid** An asteroid takes a parabolic path with Earth as its focus. It is about 6,000,000 miles from Earth at its closest approach. Write the polar equation of the path of the asteroid with its vertex at  $\theta = \pi/2$ . Find the distance between the asteroid and Earth when  $\theta = -\pi/3$ .

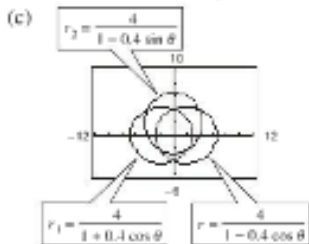
### Exploration

**True or False?** In Exercises 115–117, determine whether the statement is true or false. Justify your answer.

115. The graph of  $\frac{1}{4}x^2 - y^4 = 1$  is a hyperbola.
116. Only one set of parametric equations can represent the line  $y = 3 - 2x$ .
117. There is a unique polar coordinate representation of each point in the plane.
118. **Think About It** Consider an ellipse with the major axis horizontal and 10 units in length. The number  $b$  in the standard form of the equation of the ellipse must be less than what real number? Explain the change in the shape of the ellipse as  $b$  approaches this number.
119. **Think About It** What is the relationship between the graphs of the rectangular and polar equations?
- (a)  $x^2 + y^2 = 25$ ,  $r = 5$
- (b)  $x - y = 0$ ,  $\theta = \frac{\pi}{4}$

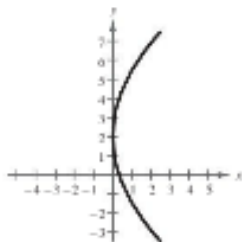
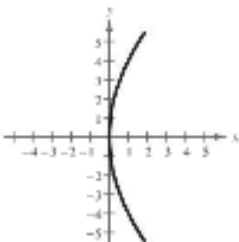
79. (a) Ellipse

(b) The given polar equation,  $r$ , has a vertical directrix to the left of the pole. The equation  $r_1$  has a vertical directrix to the right of the pole, and the equation  $r_2$  has a horizontal directrix below the pole.



**Review Exercises** (page 764)

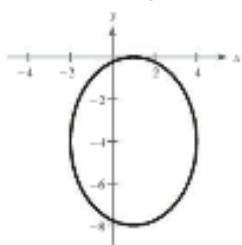
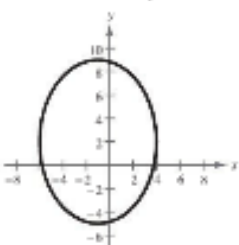
1.  $\frac{\pi}{4}$  rad,  $45^\circ$     3. 1.1071 rad,  $63.43^\circ$     5. 0.4424 rad,  $25.35^\circ$   
 7. 0.6588 rad,  $37.75^\circ$     9.  $4\sqrt{2}$     11. Hyperbola  
 13.  $y^2 = 16x$     15.  $(y - 2)^2 = 12x$



17.  $y = -4x - 2$     19.  $6\sqrt{5}$  m  
 21.  $\frac{(x - 3)^2}{25} + \frac{y^2}{16} = 1$     23.  $\frac{(x - 2)^2}{4} + \frac{(y - 1)^2}{1} = 1$

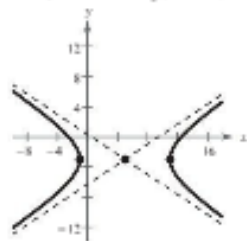
25. The foci occur 3 feet from the center of the arch.

27. Center:  $(-1, 2)$     29. Center:  $(1, -4)$   
 Vertices:  $(-1, 9), (-1, -5)$     Vertices:  $(1, 0), (1, -8)$   
 Foci:  $(-1, 2 \pm 2\sqrt{6})$     Foci:  $(1, -4 \pm \sqrt{7})$   
 Eccentricity:  $\frac{2\sqrt{6}}{7}$     Eccentricity:  $\frac{\sqrt{7}}{4}$

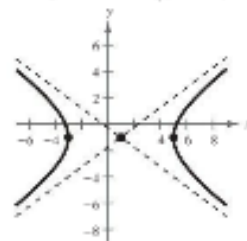


31.  $\frac{y^2}{1} - \frac{x^2}{3} = 1$     33.  $\frac{x^2}{16} - \frac{y^2}{9} = 1$

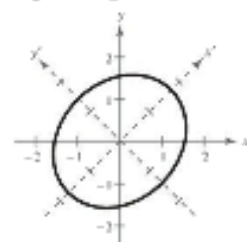
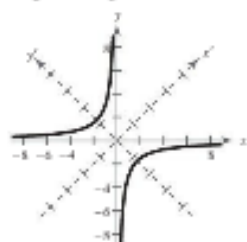
35. Center:  $(5, -3)$   
 Vertices:  $(11, -3), (-1, -3)$   
 Foci:  $(5 \pm 2\sqrt{13}, -3)$   
 Asymptotes:  $y = -3 \pm \frac{2}{3}(x - 5)$



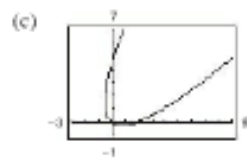
37. Center:  $(1, -1)$   
 Vertices:  $(5, -1), (-3, -1)$   
 Foci:  $(6, -1), (-4, -1)$   
 Asymptotes:  $y = -1 \pm \frac{3}{4}(x - 1)$



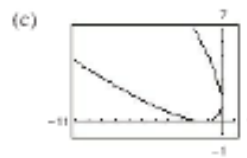
39. 72 mi    41. Hyperbola    43. Ellipse  
 45.  $\frac{(y')^2}{6} - \frac{(x')^2}{6} = 1$     47.  $\frac{(x')^2}{3} + \frac{(y')^2}{2} = 1$



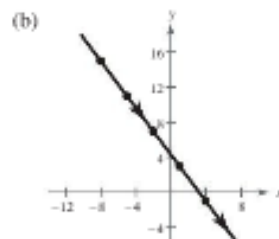
49. (a) Parabola  
 (b)  $y = \frac{24x + 40 \pm \sqrt{(24x + 40)^2 - 36(16x^2 - 30x)}}{18}$



51. (a) Parabola  
 (b)  $y = \frac{-(2x - 2\sqrt{2}) \pm \sqrt{(2x - 2\sqrt{2})^2 - 4(x^2 + 2\sqrt{2}x + 2)}}{2}$

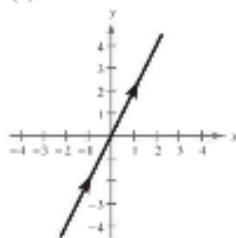


53. (a)
- |     |    |    |    |   |    |
|-----|----|----|----|---|----|
| $r$ | -2 | -1 | 0  | 1 | 2  |
| $x$ | -8 | -5 | -2 | 1 | 4  |
| $y$ | 15 | 11 | 7  | 3 | -1 |

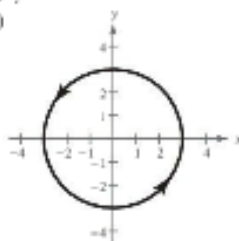




55. (a)


 (b)  $y = 2x$ 

59. (a)


 61. (a)  $x = t, y = 2t + 3$  (b)  $x = t - 1, y = 2t + 1$   
 (c)  $x = 3 - t, y = 9 - 2t$ 

 63. (a)  $x = t, y = t^2 + 3$  (b)  $x = t - 1, y = t^2 - 2t + 4$   
 (c)  $x = 3 - t, y = t^2 - 6t + 12$ 

 65. (a)  $x = t, y = 2t^2 + 2$  (b)  $x = t - 1, y = 2t^2 - 4t + 4$   
 (c)  $x = 3 - t, y = 2t^2 - 12t + 20$ 

67.


 $\left(2, -\frac{7\pi}{4}\right), \left(-2, \frac{5\pi}{4}\right)$ 

69.


 $(7, 1.05), (-7, -2.09)$ 

 71.  $\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)$  73.  $\left(-\frac{3\sqrt{2}}{2}, \frac{3\sqrt{2}}{2}\right)$  75.  $\left(1, \frac{\pi}{2}\right)$ 

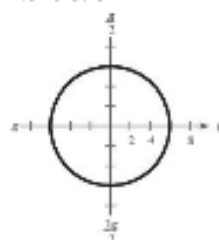
 77.  $(2\sqrt{13}, 0.9828)$  79.  $r = 9$  81.  $r = 6 \sin \theta$ 

 83.  $r^2 = 10 \csc 2\theta$  85.  $x^2 + y^2 = 25$ 

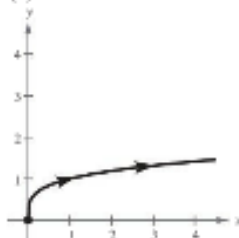
 87.  $x^2 + y^2 = 3x$  89.  $x^2 + y^2 = y^2/3$ 

 91. Symmetry:  $\theta = \frac{\pi}{2}$ , polar axis, pole

 Maximum value of  $|r|$ :  $|r| = 6$  for all values of  $\theta$ 

 No zeros of  $r$ 


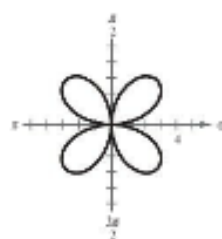
57. (a)


 (b)  $y = \sqrt[4]{x}$ 

 (b)  $x^2 + y^2 = 9$ 

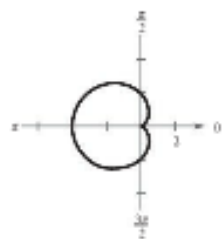
 93. Symmetry:  $\theta = \frac{\pi}{2}$ , polar axis, pole

 Maximum value of  $|r|$ :  $|r| = 4$  when  $\theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$ 

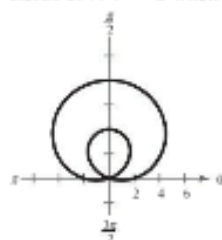
 Zeros of  $r$ :  $r = 0$  when  $\theta = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}$ 


95. Symmetry: polar axis

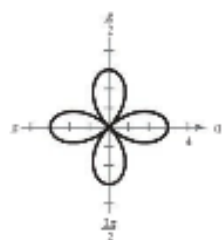
 Maximum value of  $|r|$ :  $|r| = 4$  when  $\theta = 0$ 

 Zeros of  $r$ :  $r = 0$  when  $\theta = \pi$ 

 97. Symmetry:  $\theta = \frac{\pi}{2}$ 

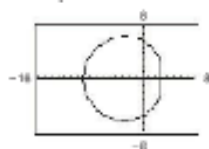
 Maximum value of  $|r|$ :  $|r| = 8$  when  $\theta = \frac{\pi}{2}$ 

 Zeros of  $r$ :  $r = 0$  when  $\theta = 3.4814, 5.9433$ 

 99. Symmetry:  $\theta = \frac{\pi}{2}$ , polar axis, pole

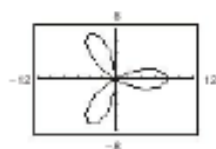
 Maximum value of  $|r|$ :  $|r| = 3$  when  $\theta = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}$ 

 Zeros of  $r$ :  $r = 0$  when  $\theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$ 


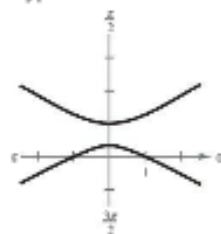
101. Limaçon



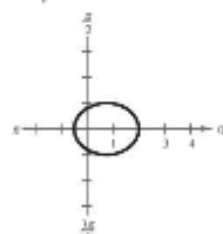
103. Rose curve



105. Hyperbola



107. Ellipse



109.  $r = \frac{4}{1 - \cos \theta}$       111.  $r = \frac{5}{3 - 2 \cos \theta}$

113.  $r = \frac{7961.93}{1 - 0.937 \cos \theta}$ ; 10,980.11 mi

115. False. The equation of a hyperbola is a second-degree equation.

117. False.  $(2, \pi/4)$ ,  $(-2, 5\pi/4)$ , and  $(2, 9\pi/4)$  all represent the same point.

119. (a) The graphs are the same.      (b) The graphs are the same.

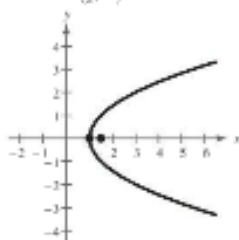
**Chapter Test** (page 767)

1.  $0.3805$  rad,  $21.8^\circ$       2.  $0.8330$  rad,  $47.7^\circ$       3.  $\frac{7\sqrt{2}}{2}$

4. Parabola:  $y^2 = 2(x - 1)$

Vertex:  $(1, 0)$

Focus:  $(\frac{3}{2}, 0)$



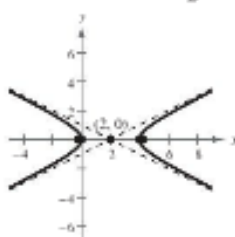
5. Hyperbola:  $\frac{(x - 2)^2}{4} - y^2 = 1$

Center:  $(2, 0)$

Vertices:  $(0, 0)$ ,  $(4, 0)$

Foci:  $(2 \pm \sqrt{5}, 0)$

Asymptotes:  $y = \pm \frac{1}{2}(x - 2)$

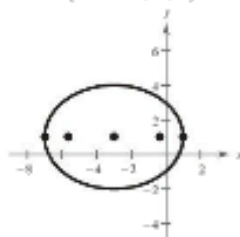


6. Ellipse:  $\frac{(x + 3)^2}{16} + \frac{(y - 1)^2}{9} = 1$

Center:  $(-3, 1)$

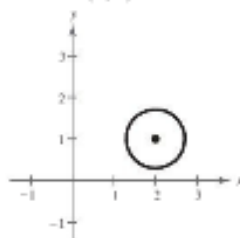
Vertices:  $(1, 1)$ ,  $(-7, 1)$

Foci:  $(-3 \pm \sqrt{7}, 1)$



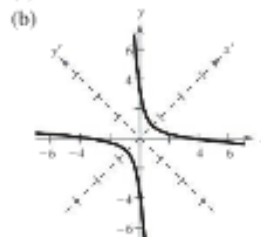
7. Circle:  $(x - 2)^2 + (y - 1)^2 = \frac{1}{2}$

Center:  $(2, 1)$

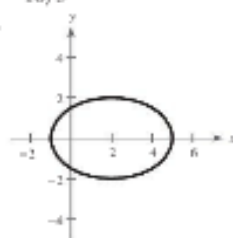


8.  $(x - 2)^2 = \frac{4}{3}(y + 3)$       9.  $\frac{y^2}{2/5} - \frac{x^2}{18/5} = 1$

10. (a)  $45^\circ$



11.



$\frac{(x - 2)^2}{9} + \frac{y^2}{4} = 1$

12. (a)  $x = t$ ,  $y = 3 - t^2$

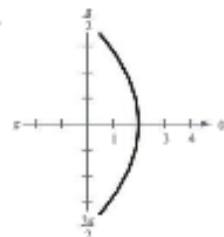
(b)  $x = t - 2$ ,  $y = -t^2 + 4t - 1$

13.  $(\sqrt{3}, -1)$

14.  $(2\sqrt{2}, \frac{7\pi}{4})$ ,  $(-2\sqrt{2}, \frac{3\pi}{4})$ ,  $(2\sqrt{2}, -\frac{\pi}{4})$

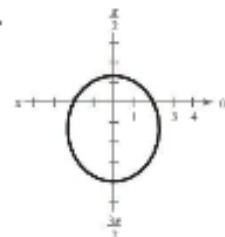
15.  $r = 3 \cos \theta$

16.



Parabola

17.



Ellipse

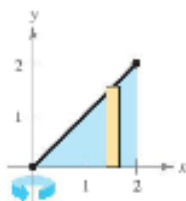


## 7.3 Exercises

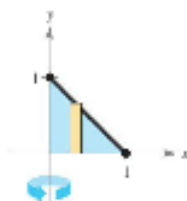
See CalcChat.com for tutorial help and worked-out solutions to odd-numbered exercises.

**Finding the Volume of a Solid** In Exercises 1–14, use the shell method to set up and evaluate the integral that gives the volume of the solid generated by revolving the plane region about the  $y$ -axis.

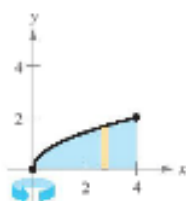
1.  $y = x$



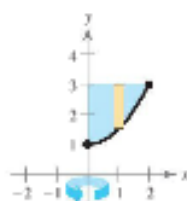
2.  $y = 1 - x$



3.  $y = \sqrt{x}$



4.  $y = \frac{1}{2}x^2 + 1$



5.  $y = \frac{1}{4}x^2$ ,  $y = 0$ ,  $x = 4$

6.  $y = \frac{1}{2}x^3$ ,  $y = 0$ ,  $x = 3$

7.  $y = x^2$ ,  $y = 4x - x^2$

8.  $y = 9 - x^2$ ,  $y = 0$

9.  $y = 4x - x^2$ ,  $x = 0$ ,  $y = 4$

10.  $y = x^{3/2}$ ,  $y = 8$ ,  $x = 0$

11.  $y = \sqrt{x-2}$ ,  $y = 0$ ,  $x = 4$

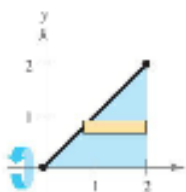
12.  $y = -x^2 + 1$ ,  $y = 0$

13.  $y = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}$ ,  $y = 0$ ,  $x = 0$ ,  $x = 1$

14.  $y = \begin{cases} \frac{\sin x}{x}, & x > 0 \\ 1, & x = 0 \end{cases}$ ,  $y = 0$ ,  $x = 0$ ,  $x = \pi$

**Finding the Volume of a Solid** In Exercises 15–22, use the shell method to set up and evaluate the integral that gives the volume of the solid generated by revolving the plane region about the  $x$ -axis.

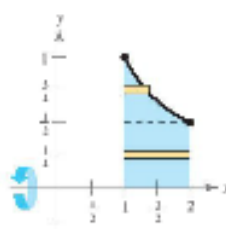
15.  $y = x$



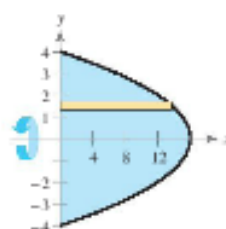
16.  $y = 1 - x$



17.  $y = \frac{1}{x}$



18.  $x + y^2 = 16$



19.  $y = x^2$ ,  $x = 0$ ,  $y = 8$

20.  $y = 4x^2$ ,  $x = 0$ ,  $y = 4$

21.  $x + y = 4$ ,  $y = x$ ,  $y = 0$

22.  $y = \sqrt{x+2}$ ,  $y = x$ ,  $y = 0$

**Finding the Volume of a Solid** In Exercises 23–26, use the shell method to find the volume of the solid generated by revolving the plane region about the given line.

23.  $y = 2x - x^2$ ,  $y = 0$ , about the line  $x = 4$

24.  $y = \sqrt{x}$ ,  $y = 0$ ,  $x = 4$ , about the line  $x = 6$

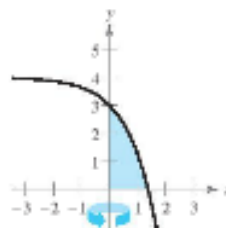
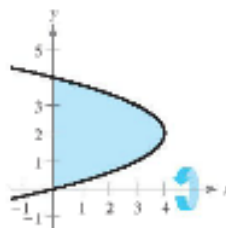
25.  $y = x^2$ ,  $y = 4x - x^2$ , about the line  $x = 4$

26.  $y = \frac{1}{3}x^3$ ,  $y = 6x - x^2$ , about the line  $x = 3$

**Choosing a Method** In Exercises 27 and 28, decide whether it is more convenient to use the disk method or the shell method to find the volume of the solid of revolution. Explain your reasoning. (Do not find the volume.)

27.  $(y-2)^2 = 4-x$

28.  $y = 4 - e^x$



**Choosing a Method** In Exercises 29–32, use the disk method or the shell method to find the volumes of the solids generated by revolving the region bounded by the graphs of the equations about the given lines.


29.  $y = x^3$ ,  $y = 0$ ,  $x = 2$

- (a) the
- $x$
- axis (b) the
- $y$
- axis (c) the line
- $x = 4$

30.  $y = \frac{10}{x^2}$ ,  $y = 0$ ,  $x = 1$ ,  $x = 5$

- (a) the
- $x$
- axis (b) the
- $y$
- axis (c) the line
- $y = 10$

31.  $x^{1/2} + y^{1/2} = a^{1/2}$ ,  $x = 0$ ,  $y = 0$   
 (a) the  $x$ -axis (b) the  $y$ -axis (c) the line  $x = a$
32.  $x^{2/3} + y^{2/3} = a^{2/3}$ ,  $a > 0$  (hypocycloid)  
 (a) the  $x$ -axis (b) the  $y$ -axis

 **Finding the Volume of a Solid** In Exercises 33–36, (a) use a graphing utility to graph the plane region bounded by the graphs of the equations, and (b) use the integration capabilities of the graphing utility to approximate the volume of the solid generated by revolving the region about the  $y$ -axis.

33.  $x^{4/3} + y^{4/3} = 1$ ,  $x = 0$ ,  $y = 0$ , first quadrant
34.  $y = \sqrt{1 - x^2}$ ,  $y = 0$ ,  $x = 0$
35.  $y = \sqrt[3]{(x - 2)^2(x - 6)^2}$ ,  $y = 0$ ,  $x = 2$ ,  $x = 6$
36.  $y = \frac{2}{1 + e^{1/x}}$ ,  $y = 0$ ,  $x = 1$ ,  $x = 3$

### WRITING ABOUT CONCEPTS

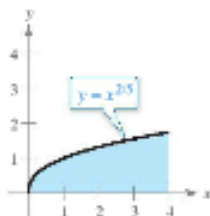
37. **Representative Rectangles** Consider a solid that is generated by revolving a plane region about the  $y$ -axis. Describe the position of a representative rectangle when using (a) the shell method and (b) the disk method to find the volume of the solid.
38. **Describing Cylindrical Shells** Consider the plane region bounded by the graphs of  
 $y = k$ ,  $y = 0$ ,  $x = 0$ , and  $x = b$   
 where  $k > 0$  and  $b > 0$ . What are the heights and radii of the cylinders generated when this region is revolved about (a) the  $x$ -axis and (b) the  $y$ -axis?

**Comparing Integrals** In Exercises 39 and 40, give a geometric argument that explains why the integrals have equal values.

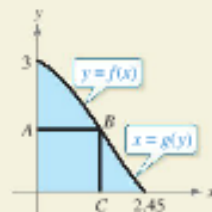
$$39. \pi \int_1^5 (x - 1) dx = 2\pi \int_0^2 y[5 - (y^2 + 1)] dy$$

$$40. \pi \int_0^2 [16 - (2y)^2] dy = 2\pi \int_0^4 x\left(\frac{x}{2}\right) dx$$

41. **Comparing Volumes** The region in the figure is revolved about the indicated axes and line. Order the volumes of the resulting solids from least to greatest. Explain your reasoning.  
 (a)  $x$ -axis (b)  $y$ -axis (c)  $x = 4$



**HOW DO YOU SEE IT?** Use the graph to answer the following.



- (a) Describe the figure generated by revolving segment  $AB$  about the  $y$ -axis.
- (b) Describe the figure generated by revolving segment  $BC$  about the  $y$ -axis.
- (c) Assume the curve in the figure can be described as  $y = f(x)$  or  $x = g(y)$ . A solid is generated by revolving the region bounded by the curve,  $y = 0$ , and  $x = 0$  about the  $y$ -axis. Set up integrals to find the volume of this solid using the disk method and the shell method. (Do not integrate.)

**Analyzing an Integral** In Exercises 43–46, the integral represents the volume of a solid of revolution. Identify (a) the plane region that is revolved and (b) the axis of revolution.

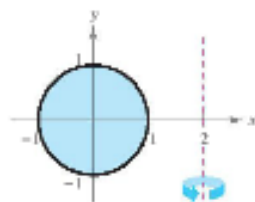
$$43. 2\pi \int_0^2 x^3 dx$$

$$44. 2\pi \int_0^1 y - y^{3/2} dy$$

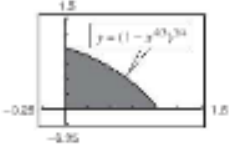
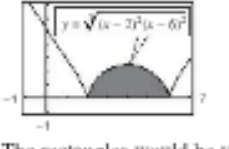
$$45. 2\pi \int_0^6 (y + 2)\sqrt{6 - y} dy$$

$$46. 2\pi \int_0^1 (4 - x)e^x dx$$

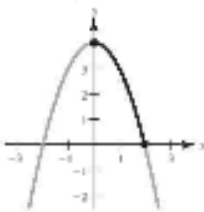
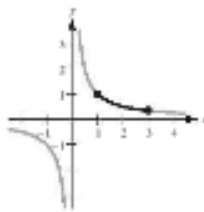
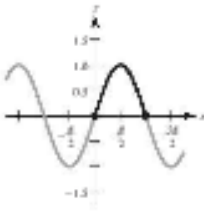

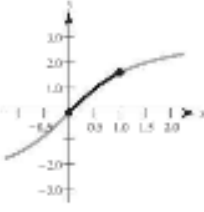
47. **Machine Part** A solid is generated by revolving the region bounded by  $y = \frac{1}{2}x^2$  and  $y = 2$  about the  $y$ -axis. A hole, centered along the axis of revolution, is drilled through this solid so that one-fourth of the volume is removed. Find the diameter of the hole.
48. **Machine Part** A solid is generated by revolving the region bounded by  $y = \sqrt{9 - x^2}$  and  $y = 0$  about the  $y$ -axis. A hole, centered along the axis of revolution, is drilled through this solid so that one-third of the volume is removed. Find the diameter of the hole.
49. **Volume of a Torus** A torus is formed by revolving the region bounded by the circle  $x^2 + y^2 = 1$  about the line  $x = 2$  (see figure). Find the volume of this “doughnut-shaped” solid. (*Hint:* The integral  $\int_{-1}^1 \sqrt{1 - x^2} dx$  represents the area of a semicircle.)



## Section 7.3 (page 462)

1.  $2\pi \int_0^2 x^2 dx = \frac{16\pi}{3}$     3.  $2\pi \int_0^4 x\sqrt{x} dx = \frac{128\pi}{5}$   
 5.  $2\pi \int_0^4 \frac{1}{4}x^2 dx = 32\pi$     7.  $2\pi \int_0^2 x(4x - 2x^2) dx = \frac{16\pi}{3}$   
 9.  $2\pi \int_0^2 x(x^2 - 4x + 4) dx = \frac{8\pi}{3}$   
 11.  $2\pi \int_2^4 x\sqrt{x-2} dx = \frac{128\pi}{15}\sqrt{2}$   
 13.  $2\pi \int_0^1 x\left(\frac{1}{\sqrt{2\pi}}e^{-x^2/2}\right) dx = \sqrt{2\pi}\left(1 - \frac{1}{\sqrt{e}}\right) \approx 0.986$   
 15.  $2\pi \int_0^2 y(2-y) dy = \frac{8\pi}{3}$   
 17.  $2\pi \left[ \int_0^{1/2} y dy + \int_{1/2}^1 y\left(\frac{1}{y} - 1\right) dy \right] = \frac{\pi}{2}$   
 19.  $2\pi \int_0^6 y^{4/3} dy = \frac{768\pi}{7}$   
 21.  $2\pi \int_0^2 y(4-2y) dy = 16\pi/3$     23.  $8\pi$     25.  $16\pi$   
 27. Shell method; it is much easier to put  $x$  in terms of  $y$  rather than vice versa.  
 29. (a)  $128\pi/7$     (b)  $64\pi/5$     (c)  $96\pi/5$   
 31. (a)  $\pi a^3/15$     (b)  $\pi a^3/15$     (c)  $4\pi a^3/15$   
 33. (a)  (b) 1.506  
 35. (a)  (b) 187.25  
 37. (a) The rectangles would be vertical.  
 (b) The rectangles would be horizontal.  
 39. Both integrals yield the volume of the solid generated by revolving the region bounded by the graphs of  $y = \sqrt{x-1}$ ,  $y = 0$ , and  $x = 5$  about the  $x$ -axis.  
 41. a, c, b  
 43. (a) Region bounded by  $y = x^2$ ,  $y = 0$ ,  $x = 0$ ,  $x = 2$   
 (b) Revolved about the  $y$ -axis  
 45. (a) Region bounded by  $x = \sqrt{6-y}$ ,  $y = 0$ ,  $x = 0$   
 (b) Revolved about  $y = -2$   
 47. Diameter =  $2\sqrt{4-2\sqrt{3}} \approx 1.464$     49.  $4\pi^2$   
 51. (a) Proof    (b) (i)  $V = 2\pi$     (ii)  $V = 6\pi^2$     53. Proof  
 55. (a)  $R_1(n) = \pi/(n+1)$     (b)  $\lim_{n \rightarrow \infty} R_1(n) = 1$   
 (c)  $V = \pi ab^{n+2}[n/(n+2)]$ ;  $R_2(n) = n/(n+2)$   
 (d)  $\lim_{n \rightarrow \infty} R_2(n) = 1$   
 (e) As  $n \rightarrow \infty$ , the graph approaches the line  $x = b$ .  
 57. (a) and (b) About 121,475 ft<sup>3</sup>    59.  $c = 2$   
 61. (a)  $64\pi/3$     (b)  $2048\pi/35$     (c)  $8192\pi/105$

## Section 7.4 (page 473)

1. (a) and (b) 17    3.  $\frac{5}{2}$     5.  $\frac{2}{3}(2\sqrt{2}-1) \approx 1.219$   
 7.  $5\sqrt{3} - 2\sqrt{2} \approx 8.352$     9. 309.3195  
 11.  $\ln\left(\frac{\sqrt{2}+1}{\sqrt{2}-1}\right) \approx 1.763$   
 13.  $\frac{1}{2}(e^2 - 1/e^2) \approx 3.627$     15.  $\frac{\pi}{3}$   
 17. (a)  (b)  $\int_0^2 \sqrt{1+4x^2} dx$   
 (c) About 4.647  
 19. (a)  (b)  $\int_1^2 \sqrt{1+\frac{1}{x^4}} dx$   
 (c) About 2.147  
 21. (a)  (b)  $\int_0^{\pi} \sqrt{1+\cos^2 x} dx$   
 (c) About 3.820  
 23. (a)  (b)  $\int_0^2 \sqrt{1+e^{-2x}} dy$   
 (c) About 2.221  
 25. (a)  (b)  $\int_0^1 \sqrt{1+\left(\frac{2}{1+x^2}\right)^2} dx$   
 (c) About 1.871  
 27. b  
 29. (a) 64.125    (b) 64.525    (c) 64.666    (d) 64.672  
 31.  $20[\sinh 1 - \sinh(-1)] \approx 47.0$  m    33. About 1480  
 35.  $3 \arcsin \frac{2}{3} \approx 2.1892$   
 37.  $2\pi \int_0^3 \frac{1}{3}x^2 \sqrt{1+x^4} dx = \frac{\pi}{9}(82\sqrt{82}-1) \approx 258.85$   
 39.  $2\pi \int_1^2 \left(\frac{x^2}{6} + \frac{1}{2x}\right)\left(\frac{x^2}{2} + \frac{1}{2x^2}\right) dx = \frac{47\pi}{16} \approx 9.23$   
 41.  $2\pi \int_{-1}^0 2 dx = 8\pi \approx 25.13$   
 43.  $2\pi \int_1^8 x \sqrt{1+\frac{1}{9x^{4/3}}} dx = \frac{\pi}{27}(145\sqrt{145} - 10\sqrt{10}) \approx 199.48$   
 45.  $2\pi \int_0^2 x \sqrt{1+\frac{x^2}{4}} dx = \frac{\pi}{3}(16\sqrt{2}-8) \approx 15.318$   
 47. 14.424  
 49. A rectifiable curve is a curve with a finite arc length.